On the Analysis of Crack Interactions Using

**Weight Function Method**

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## Abstract

A method for the analysis of interaction effect on stress intensity factors of two coplanar cracks using weight function method is presented. It provides more insight and understanding of the interaction behavior of cracks. It is demonstrated that the existence of the second crack will affect the first crack on both weight function and uncracked stress distribution. Application of the weight function method to analyze two coplanar through cracks and two coplanar semi-circular surface cracks are conducted. It is shown that for a wide range of distances between neighboring cracks, most of the interaction effect comes from the contribution of the uncracked stress distribution. Therefore, for engineering applications, the interaction effect can be accounted for from the uncracked stress distribution accurately. This provides a simplified method for the crack interaction analysis.

1. **Introduction**

In welded structures, crack initiation can occur at multiple sites. During the lifetime of the component, these initiated cracks increase in size and eventually grow together forming a dominant single crack. As the adjacent cracks grow together, an interaction occurs due to their mutual effect on each other. This interaction occurs during a substantial portion of fatigue life of the components. The driving force of fatigue used in the current fatigue life calculations is the stress intensity factor. Therefore, it is necessary to investigate the interaction effect on stress intensity factors of multiple cracks.

In practice, actual flaws in welded structures can often be approximated as either through cracks or semi-elliptical surface cracks. Several researchers have conducted studies of crack interactions for through cracks and semi-elliptical surface cracks. Solutions of stress intensity factor for certain crack geometries are available. Tada, et al (1985) summarized solutions for through cracks; and O’Donghue et. al (1984, 1986), Murakami and Nemat-Nasser (1992) and Moussa (2000) provide solutions for semi-elliptical surface cracks. However, due to the inherent complexity of the problem, overly conservative assumptions are being made in current engineering approaches. For example, different interaction criteria to determine whether two neighboring cracks will interact are given the ASME Boiler and Pressure Vessel Code (1994) Section XI and the British Standard PD 6493 (1991). And in both standards an instantaneous single crack has to be assumed as soon as any interaction occurs.

The weight function method is a very powerful technique for the calculation of stress intensity factors. It provides the stress intensity factors in a load-independent way. In the current paper, a method for the analysis of interaction effect on stress intensity factors of two coplanar cracks using weight function method is presented. It is demonstrated that the existence of the second crack will affect the first crack on both weight function and uncracked stress distribution (uncracked stress distribution denotes the stress distribution on the crack plane of the first crack due to the presence of second crack). Then weight function method is applied to calculate the interaction effect on stress intensity factors of two coplanar through cracks and two semi-circular surface cracks. It is shown that for a wide range of distances between neighboring cracks, most of the interaction effect comes from the contribution of the uncracked stress distribution. Therefore, for engineering applications, the interaction effect can be accounted for from the uncracked stress distribution accurately.

1. **Theoretical Background**

Stress intensity factor for any cracked geometry depends on the crack geometry and loading conditions. For the multiple crack problems, there are much more geometry and loading combinations compared to single crack problems; they are therefore more complicated to analyze.

*The Interaction Factor*

For a multiple crack problem, the stress intensity factor for each crack is influenced by the presence of the other cracks. This influence is generally quantified in terms of the interaction factor, which is denoted by γ. The case of two co-planar cracks, *p* and *q*, under tensile load in an infinite plate is shown in Figure 1. If the stress intensity factor at the crack tip under consideration, *p*, in the presence of another crack *q* is *Kp*, and the stress intensity factor at the same crack tip of the crack existing alone under same loading condition is *Ks*, then γ is defined as

 (1)

It accounts for the effect from the existence of other cracks. For co-planar crack problems, as is intuitively expected, the effect of each crack on the other is to increase the stress intensity factor value and resulting in γ > 1.0. It is also worth noting that the interaction factors at crack tip *A* and *B* of crack *p* are different for the current problem (Figure 1). Obviously an interaction factor of 1.0 indicates that the other crack is having no effect at this point. The γ value will depend on the overall geometry of the cracked body, the geometries of the multiple cracks, the distance between the cracks, the crack tip location and the loading conditions.

*Weight Function Method*

The weight function method (Bueckner, 1970; Rice, 1972) enables the analysis of crack problems in a loading-independent way. The weight function depends only on the geometry, and is independent of the applied loads. Consider a single cracked body loaded by mode I loads *S* as shown in Figure 2a. The stress intensity factor for this problem can be obtained by the integration of the product of weight function and a stress distribution:

 (2)

where the stress distribution σ(*x*) is stress distribution on the uncracked plane under the external loads *S* (Figure 2b) and *m*(*x*, *a*) is the weight function. For any given loading condition, Eq. (2) can be used to calculate the corresponding stress intensity factor as long as the corresponding stress distribution σ(*x*) generated by this loading is used.

In the current work, the weight function method is used to calculate the stress intensity factors for multiple crack problems. Now, consider one crack in the system of two cracks *p* and *q* under far field tension as shown in Figure 3a. The stress intensity factors at the crack tip of the one crack under consideration, *p*, can be calculated as follows:

 (3)

where *mp*(*x*, *a*) is the weight function at the crack tip (either *A* or *B*) for the specific crack *p*; and σ*p*(*x*) is the stress distribution on the crack plane (or uncracked stress distribution) of crack *p*, as shown in Figure 3b. Note that when calculating this stress distribution σ*p*(*x*), the existence of the other crack, *q*, has to be considered. This stress distribution can be obtained from the analysis of the geometry containing crack *q*. Note also that, in Eq. (3), the weight function of crack *p*, *mp*(*x*, *a*), also depends on the existence of the second crack.

Now if we calculate the stress intensity factors involved in Eq. (1), *Kp* and *Ks*, using weight function method, the interaction factor γ at the crack tip can be calculated as follows:

 (4)

where σ*p*(*x*) and *mp*(*x*, *a*) are defined previously and *σ*(*x*) and *m*(*x*, *a*) are stress distributions and weigh functions for single crack problem. Note that this expression, Eq. (4), is applicable at both points *A* and *B* as long as the corresponding weight function *mp*(*x*, *a*) is used.

1. **Analysis**

From Eq. (4) it can be observed that the interaction effect between two or more cracks can be attributed to two aspects due to the existence of the other cracks: i). the difference in the weight functions between the single crack and multiple cracks, i.e. *m*(*x*, *a*) and *mp*(*x*, *a*); and ii). the difference in the stress distributions on the perspective crack plane between the single crack and multiple cracks, i.e. *σ*(*x*) and σ*p*(*x*).

These two effects can be assessed here separately by rearranging equation (4) and introducing two more factors. Eq. (4) can be rewritten as:

 (5)

where

 (6)

and

 (7)

Note that here γS represents the effect from the difference in stress distribution between σ*p*(*x*) and σ(*x*). And γW accounts for the effect from the difference in weight functions, *mp*(*x*, *a*) and *m*(*x*, *a*). When the distance between the two cracks are far enough, σ*p*(*x*) will approach σ(*x*) and *mp*(*x*, *a*) will approach *m*(*x*, *a*). In that case both γS, γW and γ will approach 1.

In the current analysis, the weight function method is applied to calculate the interaction factors for the cases of two coplanar through cracks and two coplanar semi-circular surface cracks. The γS and γW are compared to γ values and therefore contributions from the two aspects were assessed.

*Two-Coplanar Through Cracks*

For the case of two coplanar cracks with same crack length under tensile load as shown in Figure 3, let’s consider crack *p*. The interaction factors γ are available from Tada et al (1985). In the present analysis, using Eq. (6), the interaction factors γS are calculated for a wide range of distance *s.*

Note that the weight function *m*(*x*, *a*) used in Eq. (6) is the weight function for single through crack in a infinite plate. It was taken from Rice (1972) as follows:

 (8)

The stress distribution on the perspective crack plane, for single crack is the tensile stress:

 (9)

where σ0 is the far-filed applied stress. For the case of two coplanar cracks, due to the existing of the second crack *q*, the stress distribution σ*p*(*x*) would be the Westergaard solution (Anderson, 1991):

 (9)

Substituting the above equations into Eq. (6), γS is calculated. Once γS is obtained, γw can be calculated from Eq. (5) using the corresponding γ value from Tada et. al (1985). This analysis is conducted for point *A* of the current problem; the results of γS, γw and γ are presented in Table 1 illustrated in Figures 4.

From the results, it can be seen that the effect due to difference in the stress distributions γS contributes the largest portion of the γ up to *s/a* = 0.1. And γw is not significant until *s/a* is less than 0.1. Therefore, γS provides an excellent approximation for γ when the ratio of *s/a* up to 0.1 (with maximum difference less than 11.7%).

*Two-Coplanar Semi-Circular Surface Cracks*

For the case of two semi-elliptical surface cracks with same crack length under tensile load, Murakami and Nemat-Nasser (1982) and Moussa (2000) provided solutions for γ for a wide range of crack geometries. In the current analysis, the interaction factors γS are calculated for two coplanar semi-circular cracks, (a special case of semi-elliptical cracks). Tensile loads are applied at the far field. Figure 5 shows the geometry considered. The results of γS are compared to values of γ from Murakami and Nemat-Nasser (1982) and Moussa (2000).

Equation (6) is used to calculate γS. The weight function for semi-circular surface cracks *m*(*x*, *a*) is taken from Wang and Lambert (1997). The uncracked stress distribution σ(*x*) used is the same as Eq. (8) and the uncracked stress distribution σp(*x*) is the same as Eq. (9).

The analysis is conducted for point *A*. The resulting γS values for *s/a* = 0.25, 0.3, 0.5 and 1 are presented in Table 2 and illustrated in Figure 6. The results of γ from Murakami and Nemat-Nasser (1982) for *s/a* = 0.25, 0.5 or 1 and from Moussa (2000) for *s/a* = 0.3 are also presented. It can be seen that the differences between γ and γS are within 3.6%. Therefore, γS provides excellent approximation of γ for the crack geometries considered.

Through these analyses, it is demonstrated that for a wide range of distances between neighboring cracks, most of the interaction effect γ comes from γS for both through cracks and semi-circular cracks. Therefore γS provides good approximation of γ for engineering applications (up to *s/a* = 0.1). Compared to γ, γS is much easier to calculate since only the weight function for a single crack *m*(*x*, *a*) is required. The weight functions for single cracks can be found from the literature.

1. **Conclusions**

The weight function method is applied to analyze the interaction effect of multiple co-linear or co-planar cracks. It is demonstrated that the existence of the second crack will affect the first crack on both weight function and uncracked stress distribution. Analyses of coplanar through cracks and semi-circular surface cracks have shown that the effect from the latter dominates. Therefore, for engineering applications, the interaction effect can be accounted for from the uncracked stress distribution accurately. Since only the weight function for single crack is required to calculate γS, it simplifies the analysis of crack interaction calculation significantly.

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Table 1. Interaction factors γS, γW and γ for two interacting

coplanar through cracks under tension

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| s/a | γ (Tada, et. al 1985) | γS (present) | γW  (present) | (γ-γS)/γ  % |
| 0.05 | 2.215 | 1.783 | 1.243 | 19.59 |
| 0.1 | 1.795 | 1.585 | 1.132 | 11.68 |
| 0.25 | 1.414 | 1.354 | 1.043 | 4.2 |
| 0.5 | 1.229 | 1.213 | 1.012 | 1.23 |
| 0.75 | 1.154 | 1.149 | 1.004 | 0.42 |
| 1 | 1.112 | 1.111 | 1.000 | 0.24 |
| 1.25 | 1.087 | 1.087 | 1.000 | 0.0 |
| 1.5 | 1.070 | 1.070 | 1.000 | 0.0 |
| 1.75 | 1.057 | 1.057 | 1.000 | 0.0 |
| 2 | 1.048 | 1.048 | 1.000 | 0.0 |
| 3 | 1.027 | 1.027 | 1.000 | 0.0 |
| 4 | 1.018 | 1.018 | 1.000 | 0.0 |

Table 2. Interaction factors γS and γ for two interacting

coplanar semi-circular cracks under tension

|  |  |  |  |
| --- | --- | --- | --- |
| s/a | γ  Murakami and Namat-Nasser (1992),  Moussa (2000) | γS  (present) | (γ-γS)/γ  % |
| 0.25 | 1.22 | 1.2446 | 2.02 |
| 0.3 | 1.20 | 1.2208 | 1.73 |
| 0.5 | 1.15 | 1.1094 | 3.52 |
| 1 | 1.08 | 1.0875 | 0.70 |

Figure 1. Two coplanar through cracks in a infinite plate under far-field tension



Figure 2 Weight function method



Figure 3 Crack Interaction Using Weight Function Method





Figure 4 Interaction factors for two coplanar through cracks



Figure 5 Two coplanar semi-circular cracks



Figure 6 Interaction factors for two semi-circular cracks

