Analysis of crack interactions using

weight function method

Abstract

Damage tolerant design is a current approach for addressing fatigue damage in complex structures. It is known that many structures still have considerable remaining life in the presence of damage or cracks. Using the damage tolerant design approach which utilizes frequent inspections, evaluation of damage and life prediction it is possible to extend the useful safe life of many structures and components.

In many welded or riveted structures multiple cracks initiate and grow together during the life of the structure. A special case of the multiple crack problem is when cracks emanating from rivet holes of aircraft lap joints coalesce and grow into large scale damage. This situation is classified as multi site damage (MSD). In the analysis of multi site damage, stress intensity factor calculations for interacting multiple cracks are essential for evaluation of damage during life prediction calculations. However, only single crack stress intensity factor solutions by various methods are available for wide range of geometries and multiple crack stress intensity factor solutions are very limited in literature. In this work the weight function method is used to estimate the stress intensity factor of multiple cracks. This method is advantageous over other methods because stress intensity factors can be obtained efficiently for different loading conditions.

In the current research it is found that available double crack weight functions can be replaced with single crack weight functions for approximating the stress intensity factor for double crack configurations. Even though double crack stress intensity factor solution has contribution from stress distribution and the double crack weight function, the former is found to be more (95 %) and hence the approximation works. This approximation can be used to predict the mode I, mode II stress intensity factors for overlapping cracks to demonstrate magnification and shielding and to predict crack growth of interacting edge cracks emanating from rivet holes of aircraft lap joints. Hence the crack propagation life can be calculated and Multi Site Damage (MSD) in aircraft structures can be predicted for life.

Acknowledgements

I greatly acknowledge with gratitude the invaluable guidance and help provided by my Supervisor Professor Dr. Robert Bell throughout the thesis work. He guided me right from planning to the writing part of the thesis with great care.

I greatly acknowledge with gratitude the guidance given by my Co-Supervisor Professor Dr. Xin Wang. Right from inception through the shaping of thesis work, he has given me great deal of inputs without which thesis would have been an impossible one.

I acknowledge the guidance provided by Professor Dr. C. L. Tan with his BEM research code. I acknowledge the help given by Nancy Powell, Christie Egbert.

I acknowledge the guidance provided by Assistant Professor Dr. Muthu Karthikeyan for the writing part of the thesis. I acknowledge the support of my family and friends.

Table of Contents

1. Introduction

1.1) Introduction……………………………………………………………..……1

1.2) Research objectives……………………………………………………..……5

1.3) Thesis outline…………………………………………………………………6

1. Literature review

2.1) Linear elastic fracture mechanics

2.1.1) Introduction…………………………………………………...……7

2.1.2) Stress distribution around a through crack in

an infinite plate……………………………………………………9

2.1.3) Stress intensity factor………………………………………...……15

2.1.4) Fatigue life of components damaged by cracks……………….…..16

2.1.5) Limitations of linear elastic fracture mechanics…………………..17

2.2) Multiple crack problem

2.2.1) Introduction………………………………………………….……19

2.2.2) Multiple site damage………………………………………...……19

2.3) Existing solutions for interacting cracks

2.3.1) Mode I, II crack interaction………………………………….……21

2.3.2) Hybrid approach……………………………………………..……21

2.3.3) Numerical approach………………………………………….……23

2.3.4) Experimental approach ……………………………………………25

2.4) Weight function method

2.4.1) Introduction………………………………………………….……26

2.4.2) Calculation of stress intensity factor by numerical

integration………………………………………………….……27

2.4.3) Analytical weight function…………………………………..……30

2.4.4) Approximate weight function………………………………..……30

2.4.5) Petroski – Achenbach method of approximate weight

function development…………………………………….....……30

2.4.6) Analytical weight function for a through crack in an infinite

plate – mode I, mode II……………………………………..……32

2.4.7) Analytical weight function for two equal collinear through

cracks in an infinite plate..…….…………………………………32

2.4.8) Approximate weight function for cracks emanating from

holes in an infinite plate………..…………………………...……34

2.5) Boundary element method

2.5.1) Introduction………………………………………………….……34

2.5.2) Boundary Element Method (BEM)…………………………….…35

2.5.3) Crack tip element in BEM …………………………………..……38

2.5.4) Stress intensity factor calculation…………………………………39

2.5.5) 2D boundary element mesh………………………………….……40

1. Weight function analysis of crack interaction

3.1.1) Introduction….……………………………………………………41

3.1.2) Theoretical Background……………….…………………….……41

3.1.3) Current method…………….………………………………...……42

3.2) First approximation analyses

3.2.1) Interaction of two coplanar through cracks in an infinite plate under far field tension – Mode I……………………………………47

3.2.2) Two coplanar unequal through cracks - mode I…………...……..52

3.2.3) Interaction of two equal parallel through cracks in an infinite plate under far field tension - Mode I , Mode II……………………………....69

3.2.4) Interaction of two equal parallel eccentric through cracks in an infinite plate under far field tension - Mode I , Mode II……….………..74

3.3) Second approximation analyses

3.3.1) Interaction of two coplanar through cracks in an infinite plate under far field tension - Mode I……………………………………………….114

3.3.2) Interaction of two unequal coplanar through cracks in an infinite plate under far field tension - Mode I………………………….……….119

3.3.3) Interaction of two equal parallel through cracks in an infinite plate under far field tension - Mode I , Mode II……………………….……..136

3.3.3.1) Current method of weight function development for equal parallel through cracks………………………………...………..138

3.3.3.2) Two equal parallel through cracks - Mode I - Second approximation or exact weight function…………………..……143

3.3.3.3) Two equal parallel cracks - Mode II - Second approximation or exact weight function………………………..150

3.3.4) Interaction of two equal parallel eccentric through cracks in an infinite plate under far field tension - Mode I……………………….….156

3.4) Discussion of current method…………………………………..….164

1. Interaction of cracks emanating from adjacent holes in a infinite plate

4.1) Interaction between a through crack and a rivet hole…………...…168

4.2) Interaction between two edge cracks emanating from adjacent rivet

holes…………………………………………………………………….174

4.3) Interaction between two edge cracks emanating from adjacent

rivet holes with different pitch over diameter ratios… ……..………….180

1. Conclusion and recommendations

5.1) Conclusion…………………………………………………………186

5.2) Recommendations…………………………………………………189

# List of Tables

# 3.1a Two equal coplanar cracks - mode I – close tip……………………….……………51

# 3.1b Two equal coplanar cracks - mode I – far tip………………………………………51

3.2a-3.2e Two unequal collinear cracks - mode I – close tip……………………………64

3.2f-3.2j Two unequal collinear cracks - mode I – far tip…………………………….…64

3.3a Two equal parallel cracks – mode I – first approximation……………………….…73

3.3b Two equal parallel cracks – mode II – first approximation……………………...…73

3.4a-3.4e Two equal parallel cracks – mode I – Tip A .……………………...…….……88

3.5a-3.5e Two equal parallel eccentric cracks - mode I - Close tip A ………..…………95

3.6a-3.6e Two equal parallel eccentric cracks - mode II - Close tip A…………………101

3.7a-37e Two equal parallel eccentric through cracks - mode I - Far tip B……………108

3.8a-3.8c Two equal parallel eccentric cracks - mode I - Close tip A……………….…113

3.9a-3.9b Interaction of two coplanar through cracks in an infinite plate under far field tension - mode I…………………………………………………………………….…..118

3.10a-3.10e Interaction of two unequal coplanar through cracks in an infinite plate under far field tension - mode I - Close tip A…………………………………………..……..131

3.10f-3.10j Interaction of two unequal coplanar through cracks in an infinite plate under far field tension - mode I - Far tip A……………………………………………..……..131

3.11 Two equal parallel through cracks - mode I - Second approximation or exact weight function…………………………………………………………………………..……..144

3.12 Two equal parallel cracks - Mode I - Exact weight function………………...……147

3.13 Two equal parallel cracks - Mode I - Verification test case………………..……..149

3.14 Weight function parameters m1,m2 - mode II ……………………………..……..151

3.15 Two equal parallel cracks- mode II - Second approximation……………………..152

3.16 Two equal parallel cracks - mode II - Verification test case………………….…..155

3.17 Two equal parallel eccentric cracks - Close tip A…………………………….…..163

4.1a Rivet hole and a crack - mode I - first approximation - Close tip………………....173

4.1b Rivet hole and a crack - mode I - first approximation - Far tip…………………...173

4.2a Two collinear double edge cracks - mode I - first approximation - Close tip…….179

4.2b Two collinear double edge cracks - mode I - first approximation - Far tip…….…179

* 1. Two collinear double edge cracks - mode I - first approximation - Close tip

P/d = 1.25 to 3……………………………………………………………………..183

# List of figures

* 1. Aloha airlines accident………………………………………………………………3

2.1 Three modes of fracture………………………………………………………..……8

2.2 Through crack under far field tension………………………………………….……9

2.3 Griffith-Inglis mode I through crack in an infinite plate……………………..……11

2.4 σyy stress field around a through crack……………………………………….……13

2.5 stress field around a through crack………………………………………..……14

2.6 Fatigue crack growth of a component………………………………………...……16

2.7 Plane strain and plane stress condition……………………………………….……18

2.8 Multi-Site Damage…………………………………………………………………20

2.9 Weight function method……………………………………………………...……27

2.10 Numerical integration algorithm……………………………………………..……29

2.11 Weight function for two equal collinear through cracks…………………….……32

* 1. Reference boundary conditions for BIE problem…………………………………36
  2. Traction singular quarter point crack tip element…………………………….……38
  3. Crack tip element geometry for SIF calculation……………………………...……39
  4. Sample BEM mesh…………………………………………………………...……40

3.1 Two interacting cracks (p,q) under far field tension………………………….……44

* 1. Stress distribution of first crack onto the virtual crack line of second crack………44
  2. Two equal collinear through cracks under far field tension…………………...…..47
  3. Analysis of two equal collinear through cracks under far field tension…………..48
  4. Two equal collinear through cracks - close tip interaction - mode I……………...49
  5. Two equal collinear through cracks - far tip interaction - mode I………………...50
  6. Two unequal collinear through cracks under far field tension…………………….52
  7. Analysis of two unequal collinear through cracks under far field tension.………..53
  8. –3.13 Two unequal collinear through cracks – close tip interaction- mode I………..54

3.14-3.18 Two unequal collinear through cracks – far tip interaction - mode I ………...59

3.19 Two equal parallel through cracks under far field tension………………………..69

3.20 Analysis of two equal parallel through cracks under far field tension……………70

3.21 Two equal parallel cracks – mode I – first approximation………………………..71

* 1. Two equal parallel cracks – mode II – first approximation……………………….72
  2. Two equal parallel eccentric through cracks under far field tension……………...74

3.24 Analysis of two equal parallel eccentric through cracks under far field tension.….75

3.25-3.34 Two equal parallel eccentric cracks – mode I – first approximation – close

tip 2h/a = 0 to 2.0, d/a = -2.0 to 2.66…………………………………..……78

3.35-3.39 Two equal parallel eccentric cracks – mode I – first and second approximation

– close tip - 2h/a = 0 to 2.0, d/a = 0 to 3.6………………………….…..……90

3.40-3.44 Two equal parallel eccentric cracks – mode II – first approximation – close tip

2h/a = 0 to 2.0, d/a = -2.0 to 6.0………………...……………………………96

#### 3.45-3.49 Two equal parallel eccentric through cracks - Far tips (B) – Mode I –

a/b = 0.2 – 1.0 and 2h/a = 0 - 2.0………………………………………...…103

# 3.50-3.52 Two equal parallel eccentric cracks - Close tip(A) - Mode I – x = 2.4 – 6.0 ,

y (2h/a) =2.0 – 7.0…………………………………………………………..110

* 1. Two equal coplanar through cracks under far field tension………………………114
  2. Analysis of two equal coplanar through cracks under far field tension…………..115
  3. Two equal collinear through cracks - close tip interaction - mode I…………..…116
  4. Two equal collinear through cracks - far tip interaction - mode I……………..…117
  5. Two unequal coplanar through cracks under far field tension………………..….119
  6. Analysis of two unequal coplanar through cracks under far field tension……..…120
  7. – 3.63 Two unequal collinear through cracks – close tip interaction-

mode I……….………………………………………………………….…121

3.64 – 3.68 Two unequal collinear through cracks – far tip interaction - mode I…...…126

* 1. Two equal parallel through cracks under far field tension…………………….....136
  2. Analysis of two equal parallel through cracks under far field tension…………...137
  3. Two equal parallel crack approximate weight function development…………....138
  4. Two equal parallel cracks – mode I – weight function parameters…..…………..143
  5. Two equal parallel cracks – mode I – weight function…………………………...145
  6. Two equal parallel cracks – mode I – second approximation…………………….146
  7. Two equal parallel cracks – mode I – verification test case…………………...…148
  8. Two equal parallel cracks – mode II – weight function parameters…………...…150
  9. Two equal parallel cracks – mode II – second approximation…………………...152
  10. Two equal parallel cracks – mode II – verification test case……………………..154
  11. Two equal parallel eccentric through cracks under far field tension……………..156
  12. Analysis of two equal parallel eccentric through cracks under far field tension....157
  13. – 3.85 Two equal parallel eccentric cracks close tips (A) – Mode I ,d/a =0.2 - 3.6,

y (2h/a) =0.0 - 2.0……………………...………………………………...158

* 1. Interaction between through crack and rivet hole………………………………..169
  2. Analysis of Interaction between through crack and rivet hole…………………..170
  3. Interaction between a hole and a crack – close tip – mode I…………………….171
  4. Interaction between a hole and a crack – far tip – mode I……………………….172
  5. Two equal edge cracks emanating from adjacent rivet holes……………………174
  6. Analysis of two equal edge cracks emanating from adjacent rivet holes -

close tip…………………………………………………………………………..175

* 1. Analysis of two equal edge cracks emanating from adjacent rivet holes far tip…176
  2. Interaction between two double edge cracks – close tips – mode I……………...177
  3. Interaction between two double edge cracks – far tips – mode I………………...178
  4. Two equal edge cracks emanating from adjacent rivet holes with different pitch over diameter ratios………………………………………………………………180
  5. Analysis of two equal edge cracks emanating from adjacent rivet holes with different pitch over diameter ratios………………………………………………181
  6. Two equal edge cracks emanating from adjacent rivet holes with different pitch over diameter ratios – close tips – mode I……………………………………….182

5.1 Different regions around a through crack……………………………………..…187

# Nomenclature

K, S.I.F……………………………Stress Intensity Factor

KI, KII, KIII………………………..Stress Intensity Factor for mode I, II, III

B.C.F or F.………………………..Boundary or Geometry Correction Factor

r…………………………………..Radial distance from crack tip

θ…………………………………..Angle from horizontal

σy …………………………………Yield stress

rp ………………………………….Plastic Zone radius

σ ………………………………….Far field applied nominal stress

Z(z) ………………………………Westergaard stress function

σxx ………………………………..Normal stress in x direction

σyy ………………………………..Normal stress in y direction

τxy ………………………………..Shear stress in xy direction

σ(x) ………………………………Stress distribution in the virtual crack line

m (x,a) ……………………………Weight function

ur (x,a), COD……………………..Crack opening displacement field

a…………………………………..Half crack length

………………………………..Crack growth per cycle

………………………………..Stress intensity range

C, m……………………………….Material constants

…………………………………Plastic zone radius

………………………………….Generalized modulus of elasticity

…………………………………Reference stress intensity factor

m1,m2,m3,…… .………….Weight function parameters

Ti …………………………………Traction vector

ui …………………………………Displacement vector

……………………………….. Kronecker delta

u…………………………………..Displacement along the crack line

v…………………………………..Displacement normal to the crack line

CSD………………………………Crack Sliding Displacement

……………………………..Stress distribution in the presence of first crack p

…………………………..Weight function of crack q in the presence of crack p

FEM……………………………….Finite Element Method