Analysis of crack interactions using

weight function method

Abstract

Damage tolerant design is a current approach for addressing fatigue damage in complex structures. It is known that many structures still have considerable remaining life in the presence of damage or cracks. Using the damage tolerant design approach which utilizes frequent inspections, evaluation of damage and life prediction it is possible to extend the useful safe life of many structures and components.

In many welded or riveted structures multiple cracks initiate and grow together during the life of the structure. A special case of the multiple crack problem is when cracks emanating from rivet holes of aircraft lap joints coalesce and grow into large scale damage. This situation is classified as multi site damage (MSD). In the analysis of multi site damage, stress intensity factor calculations for interacting multiple cracks are essential for evaluation of damage during life prediction calculations. However, only single crack stress intensity factor solutions by various methods are available for wide range of geometries and multiple crack stress intensity factor solutions are very limited in literature. In this work the weight function method is used to estimate the stress intensity factor of multiple cracks. This method is advantageous over other methods because stress intensity factors can be obtained efficiently for different loading conditions.

In the current research it is found that available double crack weight functions can be replaced with single crack weight functions for approximating the stress intensity factor for double crack configurations. Even though double crack stress intensity factor solution has contribution from stress distribution and the double crack weight function, the former is found to be more (95 %) and hence the approximation works. This approximation can be used to predict the mode I, mode II stress intensity factors for overlapping cracks to demonstrate magnification and shielding and to predict crack growth of interacting edge cracks emanating from rivet holes of aircraft lap joints. Hence the crack propagation life can be calculated and Multi Site Damage (MSD) in aircraft structures can be predicted for life.

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# Nomenclature

K, S.I.F……………………………Stress Intensity Factor

KI, KII, KIII………………………..Stress Intensity Factor for mode I, II, III

B.C.F or F.………………………..Boundary or Geometry Correction Factor

 r…………………………………..Radial distance from crack tip

θ…………………………………..Angle from horizontal

σy …………………………………Yield stress

rp ………………………………….Plastic Zone radius

σ ………………………………….Far field applied nominal stress

Z(z) ………………………………Westergaard stress function

σxx ………………………………..Normal stress in x direction

σyy ………………………………..Normal stress in y direction

τxy ………………………………..Shear stress in xy direction

σ(x) ………………………………Stress distribution in the virtual crack line

m (x,a) ……………………………Weight function

ur (x,a), COD……………………..Crack opening displacement field

a…………………………………..Half crack length

………………………………..Crack growth per cycle

………………………………..Stress intensity range

C, m……………………………….Material constants

…………………………………Plastic zone radius

………………………………….Generalized modulus of elasticity

…………………………………Reference stress intensity factor

m1,m2,m3,…… .………….Weight function parameters

Ti …………………………………Traction vector

ui …………………………………Displacement vector

……………………………….. Kronecker delta

u…………………………………..Displacement along the crack line

v…………………………………..Displacement normal to the crack line

CSD………………………………Crack Sliding Displacement

……………………………..Stress distribution in the presence of first crack p

…………………………..Weight function of crack q in the presence of crack p

FEM……………………………….Finite Element Method